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# ON IMPARTING TO A SOLID BODY A STABLE ROTATION MODE WITH SPECIFIED ORIENTATION OF ITS AXIS OF ROTATION

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The problem of imparting to a solid body a stable rotation mode with specified orientation of the axis of rotation in an inertial space, and with information about the body rotary motion provided by a single spin rate sensor rigidly attached to it, is considered. A control moment which would ensure the asymptotic stability of the required motion mode is sought.

**1. Statement of the problem.** We introduce the trihedron $xyz$ rigidly attached to the solid body, and denote by $\xi$ and $\eta$ the fixed unit vectors in the base $xyz$ and in the absolute space $XYZ$, respectively.

In Euler's dynamic equations

$$I\omega^{\cdot} + \omega \times I\omega = M, \ \omega = \{\omega_x, \omega_y, \omega_z\}, \ I = \operatorname{diag}\{I_x, I_y, I_z\} \qquad (1.1)$$

used here for defining the rotary motion of the solid body, we assume that $I_x < I_y < I_z$. If we select the $z$-axis as the axis of steady rotation of the body, vector $\xi = \{0, 0, 1\}$. The motion of unit vector $\eta$ in the coordinate system rigidly attached to the body is defined by the equation

$$\eta^{\cdot} + \omega \times \eta = 0 \qquad (1.2)$$

It is assumed that vector $\eta$ and the projection of angular velocity $\omega$ of the body on the sensing axis of the spin rate sensor

$$\vartheta = n \cdot \omega, \quad n = \{\alpha, \beta, \gamma\} \qquad (1.3)$$

where $n$ is a unit vector rigidly attached to the body, are measureable.

We pose the following problem. Using the information about vector $\eta$ and angular velocity $\omega$ defined by formula (1.3) determine the control moment $M = \{M_x, M_y, M_z\}$ which ensures the asymptotic stability of the stable rotation mode of the body about the axis of the greatest moment of inertia for the specified orientation of the $z$-axis in the inertial space

$$\xi = \eta, \quad \omega = \omega_*, \quad \omega_* = \{0, 0, \Omega\}, \quad \Omega = \text{const} \qquad (1.4)$$

**2. Determination of the control moment.** We represent Eq. (1.1) in the form

$$x^{\cdot} = Ax + I^{-1}(Ix \times x + M), \quad A = \begin{Vmatrix} 0 & a_1\Omega & 0 \\ a_2\Omega & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix} \qquad (2.1)$$

$$\omega = x + \omega_*, \quad a_1 = (I_y - I_z) I_x^{-1}, \quad a_2 = (I_z - I_x) I_y^{-1}$$

In new variables this problem reduces to the assurance of asymptotic stability of the equilibrium position

$$\xi = \eta, \quad x = 0.$$

We assume $y = Cx$, $C = n'$ to be the output of system (2.1).

To estimate vector $x$ by the information available to observation we use the introduced in [1] system of state estimate

$$z^{\cdot} = Az + l\,(y - Cz) + I^{-1}\,(Iz \times z + M) \tag{2.2}$$

In the system of Eqs. (1.2), (2.1), and (2.2) we pass from variables $\eta$, $x$ and $z$ to variables $\eta$, $x$, $e = x - z$

$$\eta^{\cdot} = -(x + \omega_*) \times \eta, \quad x^{\cdot} = Ax + I^{-1}(Ix \times x + M), \qquad t \geqslant t_0 \tag{2.3}$$
$$e^{\cdot} = (A - lC)e + \Psi\,(x, e), \quad \Psi\,(x, e) = I^{-1}\,(Ix \times e + Ie \times x - Ie \times e)$$

and introduce in the analysis function

$$2V = \mu\,(\eta - \xi)^2 + x'Ix + \nu \int_{t_0}^{\infty} \|\Phi\,(t_0, \tau)\,e\|^2\,d\tau, \quad \mu > 0, \quad \nu > 0 \tag{2.4}$$

in which $\Phi\,(t_0, t) = \exp\,[(A - lC)\,(t - t_0)]$ is the normalized fundamental matrix of system $\psi^{\cdot} = (A - lC)\psi$.

Since for

$$a_1 a_2 \Omega^3 \gamma\,(a_2\beta^2 - a_1\alpha^2) \neq 0$$

the eigenvalues $\lambda_i$ $(i = 1, 2, 3)$ of matrix $A - lC$ may be specified a priori [1], it is necessary that $\lambda_i$ have negative real parts

$$\operatorname{Re} \lambda_i\,(A - lC) < 0 \quad (i = 1, 2, 3) \tag{2.5}$$

If condition (2.5) is satisfied, then

$$\int_{t_0}^{\infty} \|\Phi\,(t_0, \tau)\,e\|^2\,d\tau$$

is a positive definite finction of vector $e$ [2] and, consequently, the quadratic form (2.4) is also positive definite.

On the strength of Eqs. (2.3) the derivative of function (2.4) with respect to time is of the form

$$V^{\cdot} = x'\,(-\mu\xi \times \eta + IAx + M) - \nu e'e + R\,(x, e) \tag{2.6}$$
$$R\,(x, e) = \nu e' S \Psi\,(x, e), \quad S = \int_{t_0}^{\infty} \Phi'\,(t_0, \tau)\,\Phi\,(t_0, \tau)\,d\tau$$

By selecting the control moment $M$ of the form

$$M = \mu\xi \times \eta + Kz, \tag{2.7}$$
$$K = \operatorname{diag}\ \{k_1, k_2, k_3\}$$

we can represent formula (2.6) as

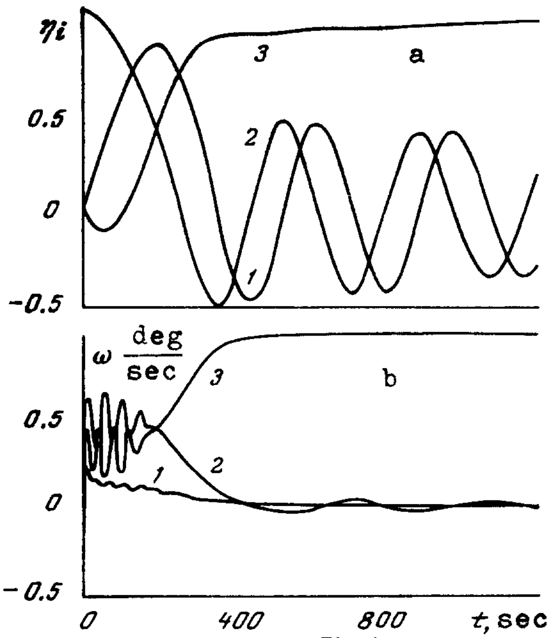

Fig. 1

$$V^{\cdot} = s'Qs + R\ (x,\ e),\qquad (2.8)$$
$$Q = {}^1/_2\ (P + P')$$
$$P = \left\| \begin{matrix} IA + K & -K \\ 0 & -\nu E \end{matrix} \right\|,\quad S = \left\| \begin{matrix} x \\ e \end{matrix} \right\|$$

where $E$ is a third order unit matrix.

The fixed sign of analytic functions is determined by the totality of terms of the lowest order in expansions of these functions [3], hence the right-hand side of formula (2.8) is, as a function of vectors $x$ and $e$, negative definite, when the quadratic form $s'Qs$ has that property.

For function $s'Qs$ to be negative definite it is necessary and sufficient that the system of inequalities

$$k_i < 0,\quad 1 + {}^1/_4\nu^{-1}k_i > 0\quad (i = 1, 2, 3)\qquad (2.9)$$
$$k_1k_2\,(1 + {}^1/_4\nu^{-1}k_1)\,(1 + {}^1/_4\nu^{-1}k_2) - {}^1/_4\Omega^2\,(I_y - I_x)^2 > 0$$

is satisfied.

When conditions (2.9) are satisfied, the right-hand side of formula (2.8) is, as a function of vector $r = \{\eta',\ x',\ e'\}$, of constant negative sign, since it is zero not only when $\xi = \eta$, and $x = e = 0$, but, also, in the set

$$N = \{r\colon\ \xi \neq \eta,\ x = 0,\ e = 0\}\qquad (2.10)$$

which we define as follows:

$$N = N_1 \cup N_2,\quad N_1 = \{r\colon \xi = -\eta,\ x = 0,\ e = 0\},\quad N_2 = \{r\colon \xi \neq \pm\eta,\ x' = 0,\ e = 0\}$$

The analysis of the first approximation derived with reference to point $N_1$ shows that $N_1$ represents an unsteady equilibrium position of system (2.3), while set $N_2$ does not contain integral trajectories of the investigated system. Hence the equilibrium position $\xi = \eta$, $x = 0$, and $e = 0$ is asymptotically stable [4].

Thus, when conditions (2.5) and (2.9) are satisfied, the control (2.7) in which vector $z$ is determined by Eq. (2.2) provides the solution of this problem in some neighborhood $G$ of the equilibrium position.

Note that the possibility of realizing the required orientation in the case of incomplete information from sensors of the body angular position occurs in the solution of the problem formulated in Sect. 1 and, also, in the problem of orientation in a rotating coordinate system [5]. In fact, the analysis of formula (2.7) shows that information about two components of vector $\eta$ (in this case $\eta_x$ and $\eta_y$) is sufficient for controlling orientation of the axis of rotation in an inertial space.

**3. Example.** Let us consider the process of imparting to a solid body whose inertia ellipsoid parameters are

$$I_x = 1.25 \cdot 10^6 \text{ kg}\cdot\text{m}^2, \quad I_y = 6{,}9 \cdot 10^6 \text{ kg}\cdot\text{m}^2, \quad I_z = 7.1 \cdot 10^6 \text{ kg}\cdot\text{m}^2$$

a stable rotation mode $\omega_x = \omega_y = 0$, and $\omega_z = \Omega = 1$ deg / sec at specified single-axis orientation in an inertial space.

Angles of the sensitivity axis of the spin rate sensor are at the same angle $n = \| 1 / \sqrt{3} \; 1 / \sqrt{3} \; 1 / \sqrt{3} \|'$ to the axes of the $xyz$ -base. Vector $l$ in system (2.2) for the estimate of the state and matrix $K$ in the control law (2.7) are, respectively,

$$l = \| -3.62 \quad -37.80 \quad 41.91 \|'$$
$$K = \operatorname{diag}\{-1.25 \cdot 10^5, \quad -6.9 \cdot 10^5, \quad -7.1 \cdot 10^5\}$$

and the weighting factors $\mu$ and $\nu$ in the quadratic form (2.4) are $\mu = 1.25 \cdot 10^2$ Nm and $\nu = 6.9 \cdot 10^6$ Nmsec$^{-1}$.

The question of selection of the initial condition for the system for the estimate of the state is decided similarly to [1].

At the instant of commencement of control of body motion the mismatch between the $z$ -axis (vector $\xi$ ) and the required direction (vector $\eta$ ) is 90 °. Initial values of remaining orientation parameters appear in Fig. 1 ( a ), where curves 1, 2, and 3 show the pattern of directional cosines $\eta_x$ $(xyz)$ in the course of control action (Fig. 1 ( a ) ) and the variation of angular velocities $\omega_x$, $\omega_y$ and $\omega_z$ of the body (Fig. 1 ( b ) ).

The analysis of the character of variation of orientation of rotation axis parameters and of the angular velocity vector $\omega$ shows that in the course of motion control the body asymptotically tends to the specified mode.

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